Machine learning in cardiovascular flows modeling: Predicting arterial blood pressure from non-invasive 4D flow MRI data using physics-informed neural networks

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Motivation

- We introduce physics-informed neural networks to solve conservation laws in graph topologies.
- We parametrize the solution of partial differential equations using deep neural networks to predict arterial pressure from MRI data of blood velocity and wall displacement.
- Our model also allows for calibrating boundary conditions of conventional flow simulators.

Method

We consider the system or normalized/non-dimensionalized equations parametrized by a neural network:

$$\frac{1}{\sigma_{t_*}}\frac{\partial \hat{A}^j}{\partial \hat{t}} + \frac{1}{\sigma^j_{x_*}}\hat{A}^j\frac{\partial \hat{u}^j}{\partial \hat{x}^j} + \frac{1}{\sigma^j_{x_*}}\hat{u}^j\frac{\partial \hat{A}^j}{\partial \hat{x}^j} = 0,$$

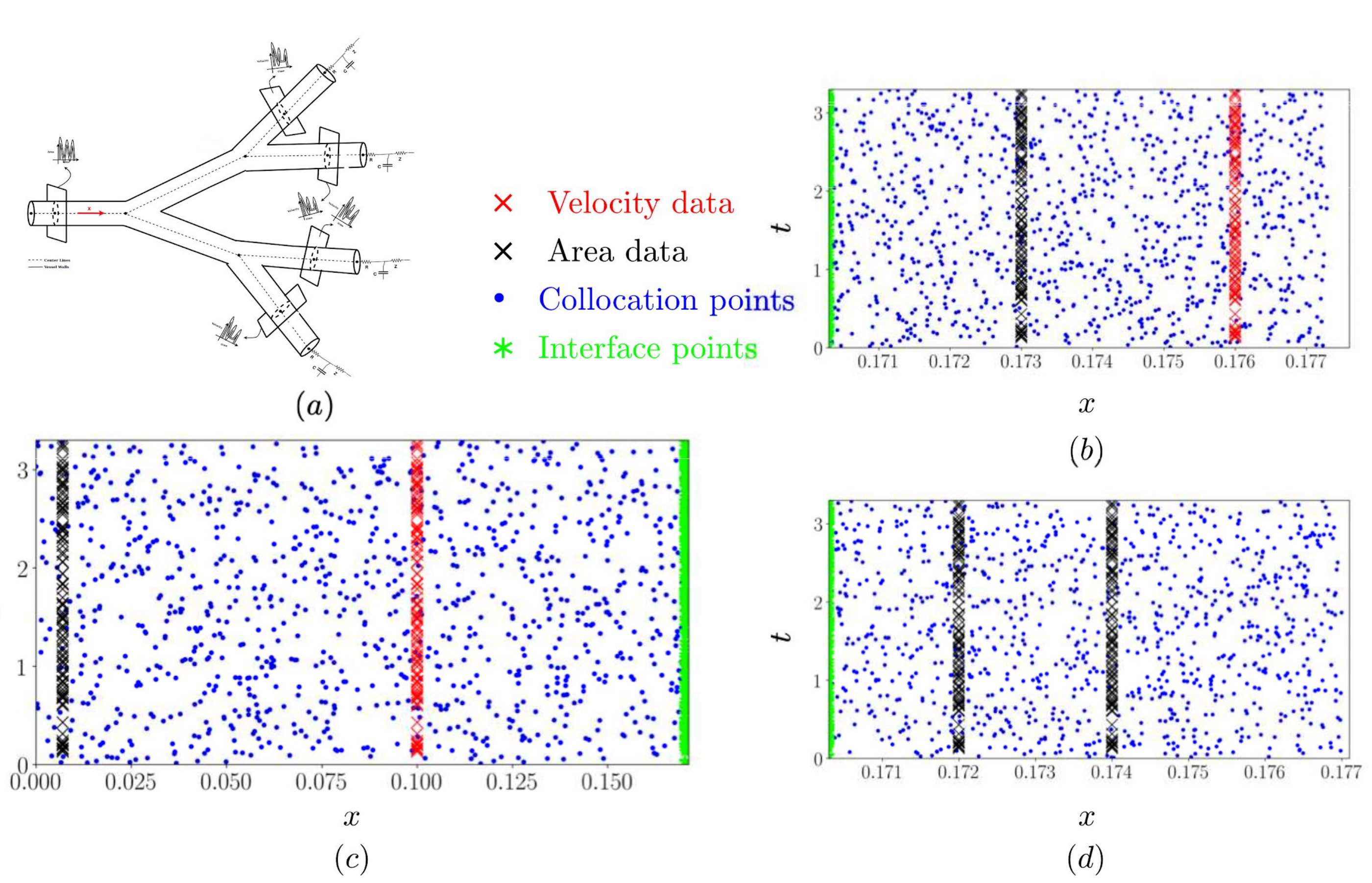
$$\frac{1}{\sigma_{t_*}}\frac{\partial \hat{u}^j}{\partial \hat{t}} + \frac{1}{\sigma^j_{x_*}}\alpha\hat{u}^j\frac{\partial \hat{u}^j}{\partial \hat{x}^j} + \frac{1}{\sigma^j_{x_*}}\hat{u}^j\frac{\partial(\alpha-1)\hat{u}^j\hat{A}^j}{\partial \hat{x}^j} + \frac{1}{\sigma^j_{x_*}}\frac{\partial \hat{p}^j}{\partial \hat{x}^j} - \frac{K_R}{LU}\frac{\hat{u}^j}{\hat{A}^j} = 0,$$

$$\hat{p}^j = \frac{1}{p_0}\left(p_{ext} + \beta\left(\sqrt{\hat{A}^j A^o} - \sqrt{A_0}\right)\right), \qquad j = 1,\dots,D.$$

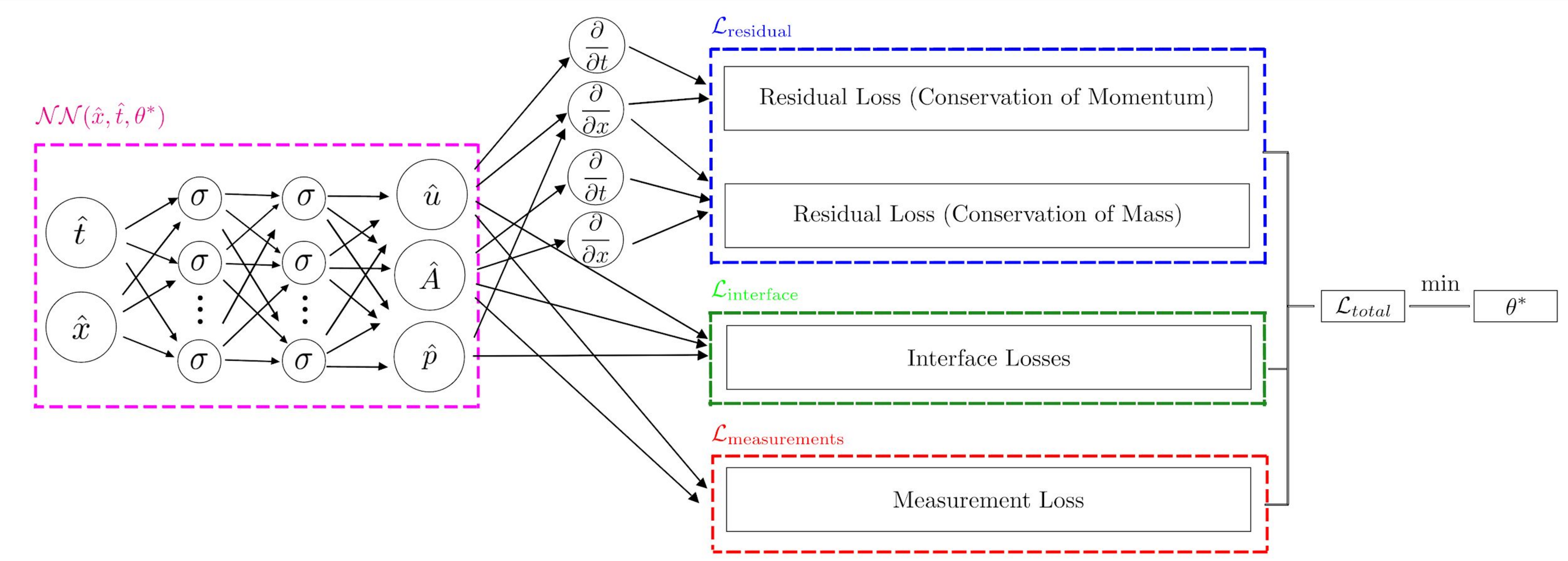
With interface boundary conditions:

$$\hat{A}_1\hat{u}_1 = \hat{A}_2\hat{u}_2 + \hat{A}_3\hat{u}_3,$$
$$\hat{p}_1 + \frac{1}{2}(\hat{u}_1)^2 = \hat{p}_2 + \frac{1}{2}(\hat{u}_2)^2,$$
$$\hat{p}_1 + \frac{1}{2}(\hat{u}_1)^2 = \hat{p}_3 + \frac{1}{2}(\hat{u}_3)^2.$$

We use velocity and wall-displacement measurements at a few cross sections to train the model (no data for the pressure is assumed).



Method



We train the neural networks by minimizing a composite loss function that aims to: fit the observed data, ensure conservation of mass and momentum, and enforce continuity at interfaces (e.g. bifurcations, junctions, etc.).

$$\mathcal{L}^j_{\text{measurements}} = \frac{1}{N^j_u}\sum_{i=1}^{N^j_u}(u^j(x_i,t_i) - u^j(x_i,t_i;\theta^j))^2 + \frac{1}{N^j_A}\sum_{i=1}^{N^j_A}(A^j(x_i,t_i) - A^j(x_i,t_i;\theta^j))^2, \qquad j = 1,\dots,D_M$$

$$\mathcal{L}^j_{\text{residual}} = \frac{1}{N^j_r}\sum_{i=1}^{N^j_r}(r^j_A(x_i,t_i;\theta^j))^2 + \frac{1}{N^j_r}\sum_{i=1}^{N^j_r}(r^j_u(x_i,t_i;\theta^j))^2 + \frac{1}{N^j_r}\sum_{i=1}^{N^j_r}(r^j_p(x_i,t_i;\theta^j))^2, \qquad j = 1,\dots,D$$

$$\begin{aligned}\mathcal{L}^k_{\text{interfaces}} = &\frac{1}{N^k_b}\sum_{i=1}^{N^k_b}(A^k_1(x_k,t_i;\theta^k_1)u^k_1(x_k,t_i;\theta^k_1) - A^k_2(x_k,t_i;\theta^k_2)u^k_2(x_k,t_i;\theta^k_2) - A^k_3(x_k,t_i;\theta^k_3)u^k_3(x_k,t_i;\theta^k_3))^2 +\\ &+ \frac{1}{N^k_b}\sum_{i=1}^{N^k_b}(p^k_1(x_k,t_i;\theta^k_1) + \frac{1}{2}u^k_1(x_k,t_i;\theta^k_1)^2 - p^k_2(x_k,t_i;\theta^k_2) - \frac{1}{2}u^k_2(x_k,t_i;\theta^k_2)^2)^2 +\\ &+ \frac{1}{N^k_b}\sum_{i=1}^{N^k_b}(p^k_1(x_k,t_i;\theta^k_1) + \frac{1}{2}u^k_1(x_k,t_i;\theta^k_1)^2 - p^k_3(x_k,t_i;\theta^k_3) - \frac{1}{2}u^k_3(x_k,t_i;\theta^k_3)^2)^2, \qquad k = 1,\dots,D_I\end{aligned}$$

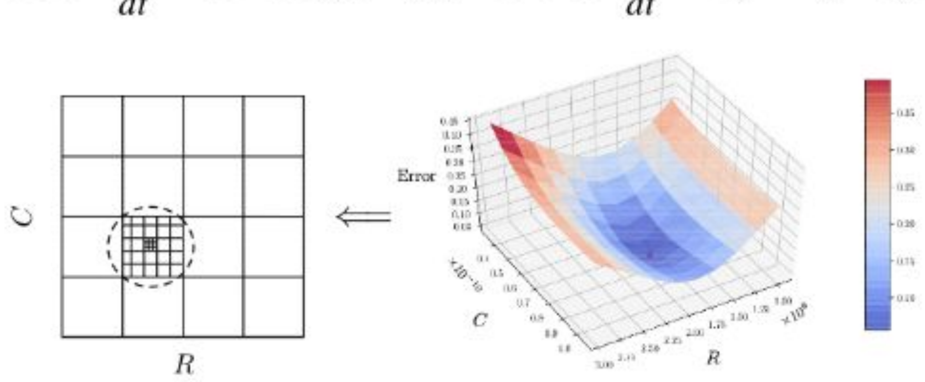
The total loss reads as

$$\mathcal{L} = \sum_{j=1}^{D_M}\mathcal{L}^j_{\text{measurements}} + \sum_{j=1}^{D}\mathcal{L}^j_{\text{residual}} + \sum_{k=1}^{D_I}\mathcal{L}^k_{\text{interfaces}}$$

and it is minimized using stochastic gradient descent.

Calibration of Windkessel Model Parameters

$$p^j + R^jC^j\frac{dp^j}{dt} - (R^j + Z^j)Q^j - p_{\text{inf}} - R^jC^jZ^j\frac{dQ^j}{dt} = 0, \qquad j = 1,\dots D_o.$$

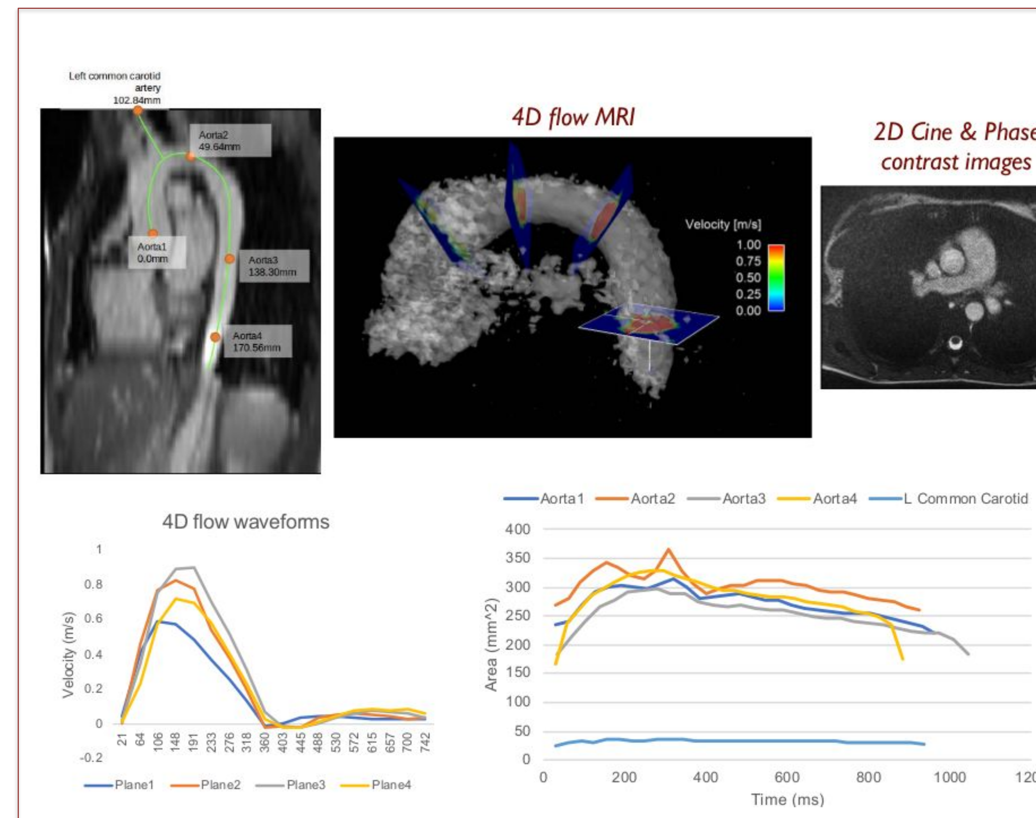


$$\text{Loss}(R^j, C^j) = \frac{1}{N_t}\sum_{m=1}^{N_t}\|\tilde{p}^j(t_{N_t}) - p^j(t_{N_t})\|^2$$

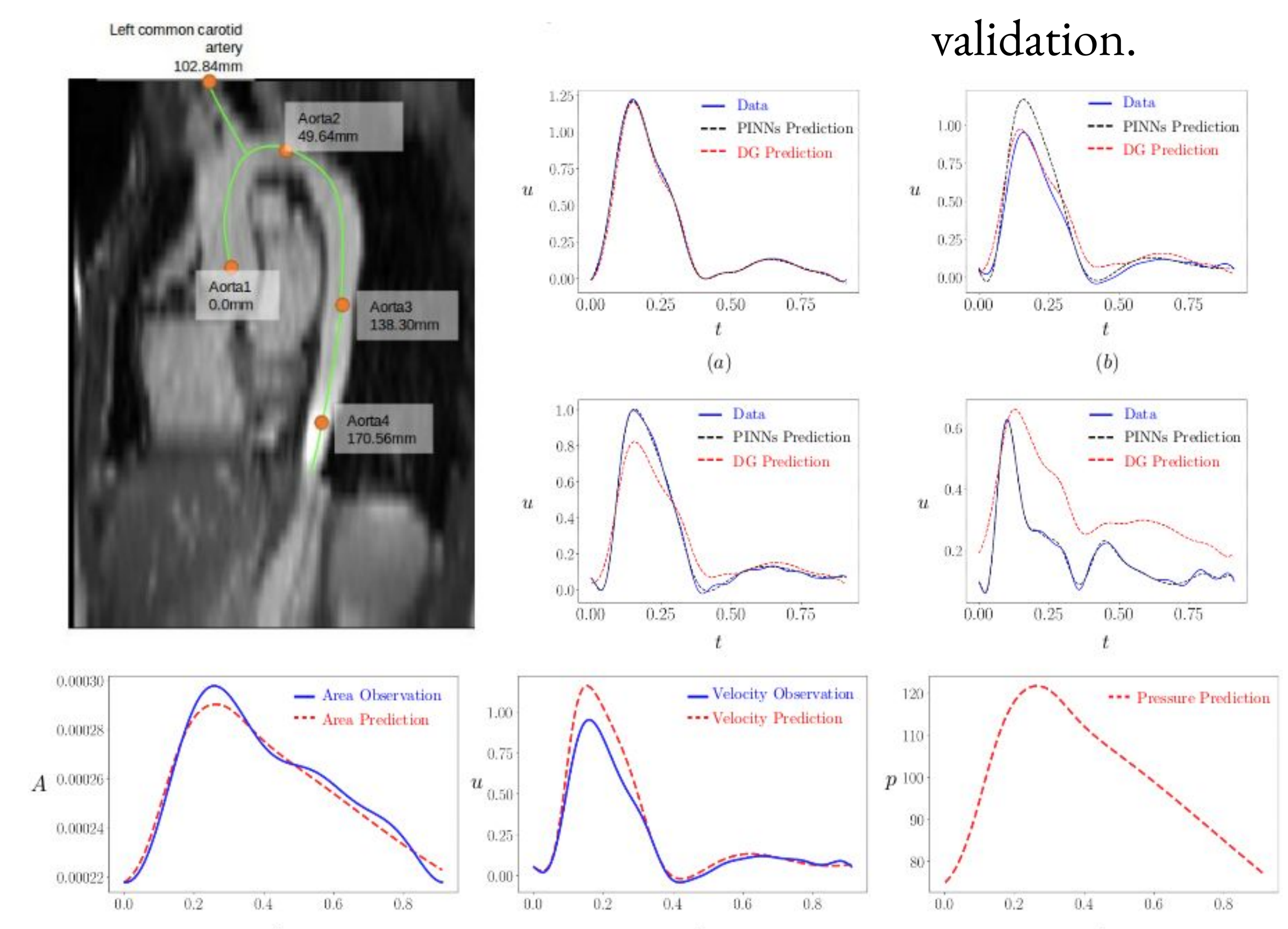
Param / # vessel	R_{Pred}	C_{Pred}	Relative error of P waveforms
#4	1.36e+09	3.78e-09	7.92e-03
#5	1.64e+09	2.22e-10	3.29e-02
#6	1.22e+09	2.96e-09	8.26e-03
#7	2.69e+09	1.84e-09	2.56e-02

Utilizing the trained neural networks to predict the pressure and volumetric flow at each temporal and spatial point, we can easily calibrate outflow boundary conditions of conventional flow simulators by estimating Windkessel model parameters using a simple and relatively cheap post-processing step.

Arterial blood pressure prediction using real noisy data



- Measurements of cross-sectional area and velocity of a healthy human subject are acquired using 4D flow MRI.
- Measurements at Aorta 1,2,4 and Carotid locations are utilized to train the model.
- Measurements at Aorta 3 are utilized for validation.



- The trained model enables physcs-based filtering of noisy clinical data and yields a reasonable predictions for the velocity, area, and pressure waves..

References

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