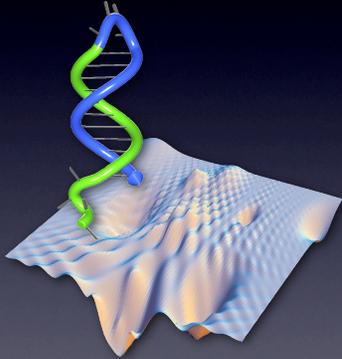


# Analysis of Coarse-Grained Nucleic Acid Free Energy Landscapes



Niles Pierce  
Erik Winfree  
Caltech

Hideo Mabuchi  
Stanford

Bernard Yurke  
Boise State

August 4, 2008  
Montreal

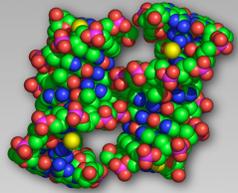
## Research Goals

Develop algorithms to map coarse-grained nucleic acid free energy landscapes – identify the kinetic and equilibrium properties that dominate experimental observables

Computational and experimental analysis of free energy landscapes with metastable states

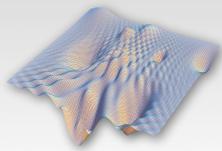
## Free Energy Landscape

### Tertiary Structure



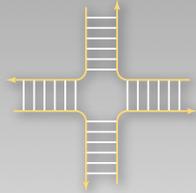
List of atomic coordinates and bonds

### Landscape



Continuous representation

### Secondary Structure



List of base pairs

### Landscape



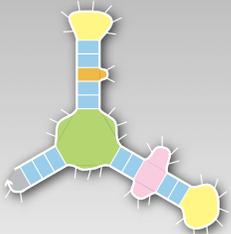
Graph representation

Each node is a secondary structure  
Each edge is an elementary kinetic step

## Thermodynamic Model

### Free Energy

Empirical loop-based free energy model for secondary structure  $s$



$$\Delta G(s) = \sum_{\text{loop} \in s} \Delta G(\text{loop})$$

Turner and SantaLucia

### Partition Function

The partition function over the ensemble of secondary structures  $\Omega$

$$Q = \sum_{s \in \Omega} e^{-\Delta G(s)/kT}$$

can be used to calculate the equilibrium probability of any secondary structure  $s \in \Omega$

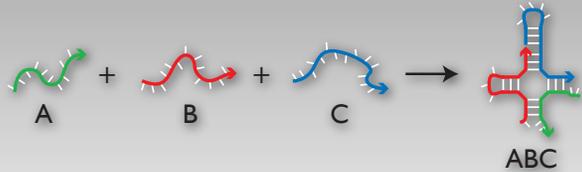
$$p(s) = \frac{1}{Q} e^{-\Delta G(s)/kT}$$

$|\Omega|$  grows exponentially with strand length

Zuker and McCaskill

## Project 1: Thermodynamic Analysis of Interacting Strands

### Partition function for a complex



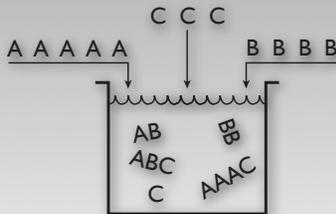
New algorithm for calculating the partition function of an unpsuedoknotted complex of multiple interacting strands

Rigorous treatment of representation and distinguishability issues that do not arise in the single-stranded case

R.M. Dirks, J.S. Bois, J.M. Schaeffer, E. Winfree, N.A. Pierce, *SIAM Rev*, 2007

## Project 1: Thermodynamic Analysis of Interacting Strands

### Concentration determination in the thermodynamic limit

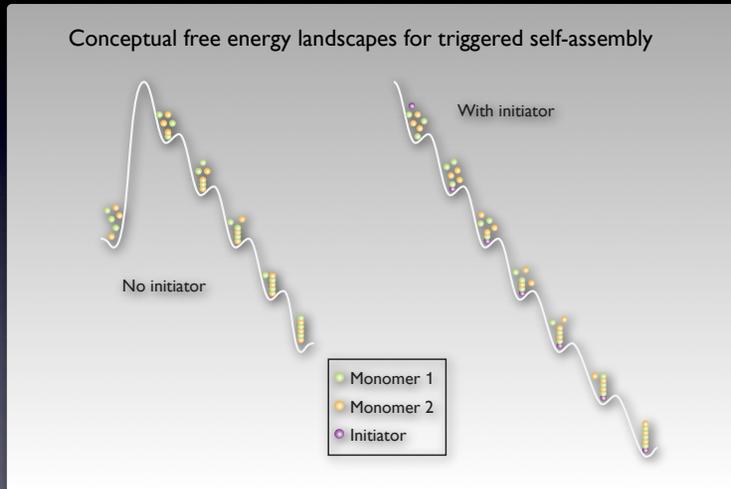


For large systems (e.g. a test tube), the equilibrium concentration of each species of complex can be obtained by solving a convex programming problem

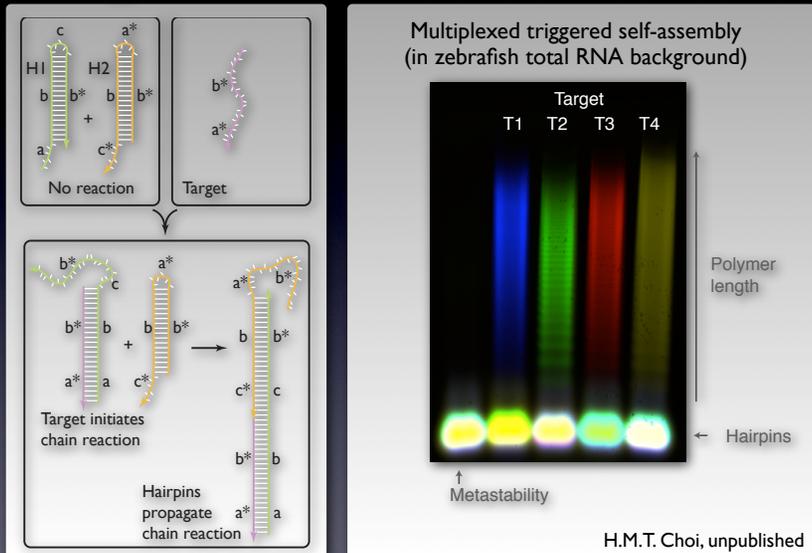
Efficient solution of the (high-dimensional) primal problem is achieved via solution of the (low-dimensional) dual problem

R.M. Dirks, J.S. Bois, J.M. Schaeffer, E. Winfree, N.A. Pierce, *SIAM Rev*, 2007

## Project 2: Probing a free energy landscape with metastable states



## Triggered self-assembly of metastable DNA hairpins



## Probing hairpin metastability

Are the hairpins metastable in the absence of initiator?

Experiment: anneal hairpins without initiator to see if they polymerize

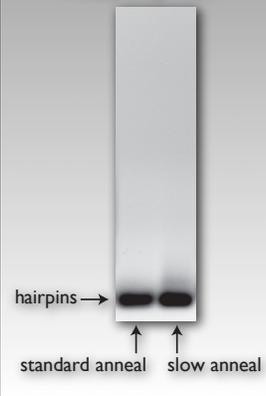
No polymerization observed with 36-hour anneal

Competing hypotheses:

Hairpins are metastable and the landscape is difficult to navigate via annealing

Hairpins are stable

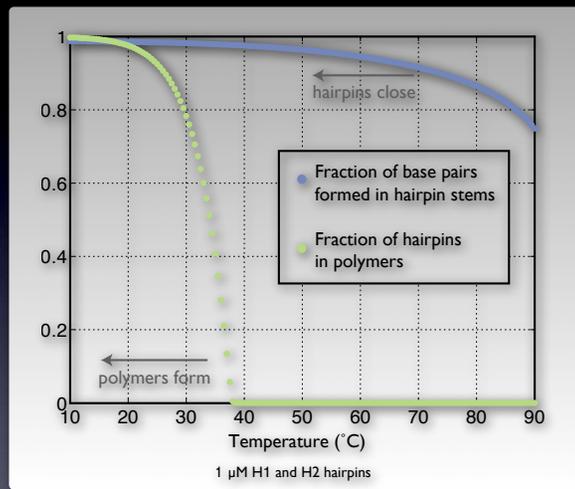
A surprising result



Standard anneal: 90s at 90°C  
cool on bench top  
Slow anneal: 95°C - 20°C  
cool 30 min/°C

V. Beck, J.S. Bois, R.M. Dirks, N.A. Pierce, in preparation

## Predicted annealing behavior



As cooling proceeds, hairpins are predicted to close before polymerization becomes energetically favorable

## Tunneling through the landscape

Trigger polymerization using an RNA initiator and then digest to obtain products with no bound initiator

Do they break up into hairpins?

No – the polymers increase in length

Supports the hypothesis that hairpins are metastable

Annealing recharges the hairpins



## Kinetic Model

Model the time-varying probability  $p_i(t)$  of secondary structure  $i$  via the master equation

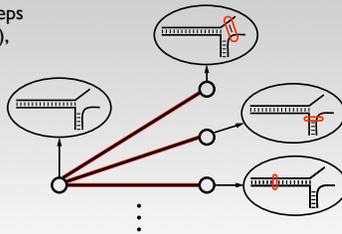
$$\frac{dp_i(t)}{dt} = \sum_{j=1}^{|\Omega|} [k_{j \rightarrow i} p_j(t) - k_{i \rightarrow j} p_i(t)]$$

N.G. van Kampen, 1992

Rate constants are positive for elementary steps (e.g., addition or deletion of a single base pair), zero otherwise

Recall,  $|\Omega|$  grows exponentially with strand length (too many states to write down)

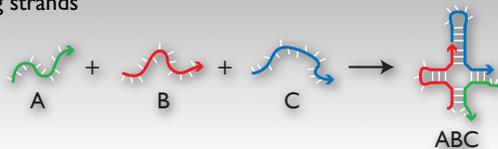
Simulate continuous-time Markov trajectories – generate move set on the fly based on current state



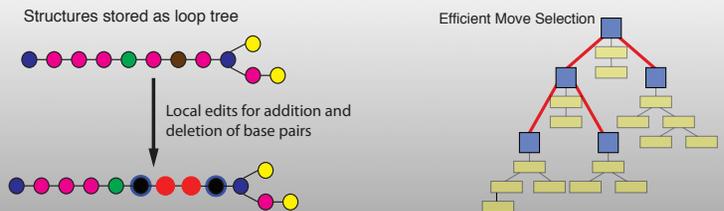
D.T. Gillespie, *J Comput Phys*, 1976  
C. Flamm, W. Fontana, I.L. Hofacker, P. Schuster, *RNA*, 2000

### Project 3: Fast Stochastic Kinetics for Interacting Strands

New algorithm for stochastic kinetic simulations of unspseudoknotted complexes of interacting strands

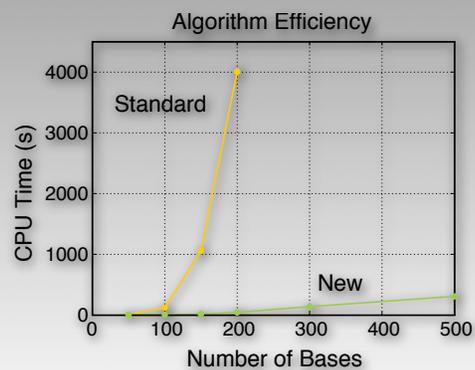


Lower complexity formulation with efficient data structures



J.M. Schaeffer, J.S. Bois, R.M. Dirks, N.A. Pierce, E. Winfree, in preparation

### Project 3: Fast Stochastic Kinetics for Interacting Strands



Fixed simulated time for random sequences from unstructured initial conditions

## Project 4: Trajectory-Based Coarse-Graining

### Goal

Discover a macrostate master equation to identify landscape features that dominate experimental measurements

### Difficulty

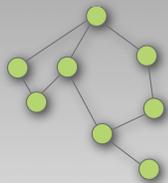
The landscape is too large to enumerate the secondary structures – cannot write down the 'microstate' model

### Approach

Use stochastic trajectories to explore the landscape (from specified initial conditions)

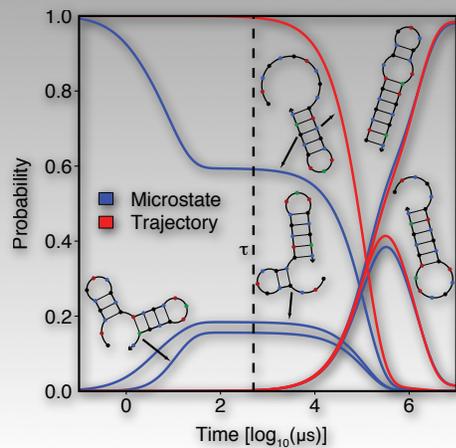
Identify transitions between macrostates by comparing occupancy distributions along trajectories (for a specified relaxation time scale  $\tau$ )

Estimate macrostate partition functions and transition rates (with confidence intervals) using trajectories



J.A. Othmer and N.A. Pierce, in preparation

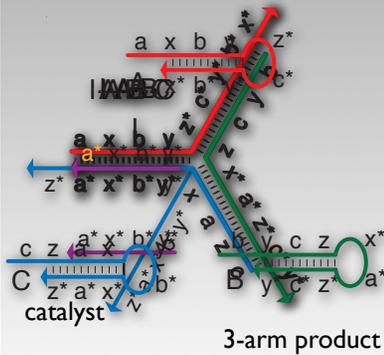
## Testing Trajectory-Based Coarse-Graining



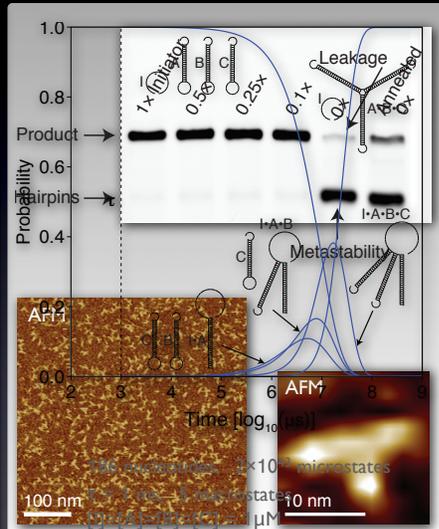
23 nucleotides  
 $2 \times 10^4$  microstates  
 $\tau = 0.5$  ms  
3 macrostates

## Trajectory-based coarse-graining example

Catalytic self-assembly of branched junctions



P.Yin, H. M.T. Choi, C.R. Calvert, N.A. Pierce, *Nature*, 2008

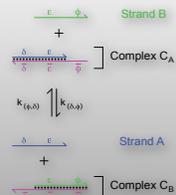


## Project 5: Quantitative characterization of barrier states in catalytic pathways

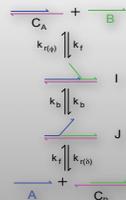
Toehold mediated strand displacement and toehold exchange are fundamental mechanisms in the design of DNA reaction pathways

Toehold sequences control reaction rates by determining barrier state energetics

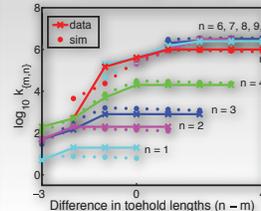
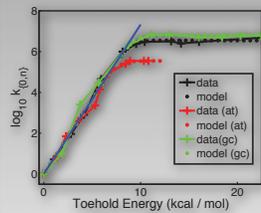
Experimentally measurable net reaction rates



Unified macrostate model across toehold length and sequence composition



Remarkable agreement of experiment and theory



D.Y. Zhang, E. Winfree, in preparation



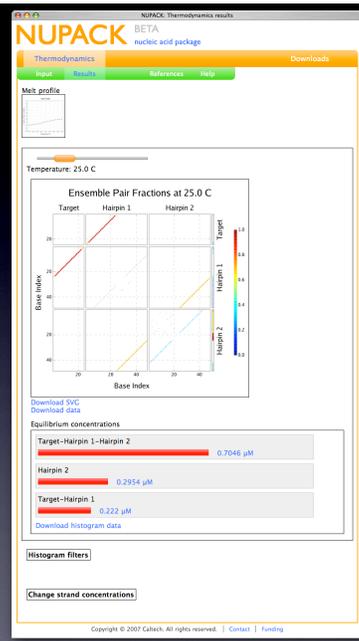
## Project 6: NUPACK Nucleic Acid Package

[www.nupack.org](http://www.nupack.org)

A growing software suite for the analysis  
and design of nucleic acid systems

Web-based interface for efficient  
hypothesis testing and easy access

Downloadable source code for  
customization and batch processing



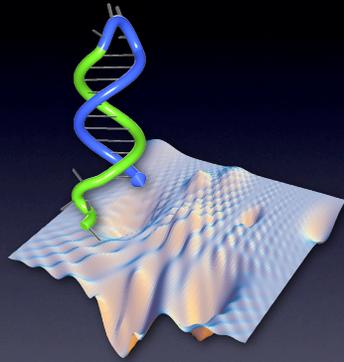
## Project 6: NUPACK Nucleic Acid Package

[www.nupack.org](http://www.nupack.org)

July 6, 2007-July 6, 2008



J.N. Zadeh, J.S. Bois, C. Steenberg, M.B. Pierce, R.M. Dirks, J.A. Othmer and N.A. Pierce, in preparation



Niles Pierce  
Erik Winfree  
Caltech

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nupack.org  
Software

NSF  
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