

## Understanding the ComplFlowDecayPlus Model

A Flow carries an inflow concentration,  $C_{in}$ , into a one compartment model with a given Volume.  $C_{in}$  is constantly and instantaneously well mixed becoming  $C$ , the concentration in the compartment.  $C$  empties out of the compartment and is designated  $C_{out}$ .  $G$  is a clearance rate.

For a constant concentration of inflowing material the analytic solution is given.

The ComplFlowDecay model combines the ComplDecay model with the ComplFlow model. The governing ordinary differential equation is given as

$$dC/dt = (F/V) \cdot (C_{in} - C) - (G/V) \cdot C$$

with initial condition

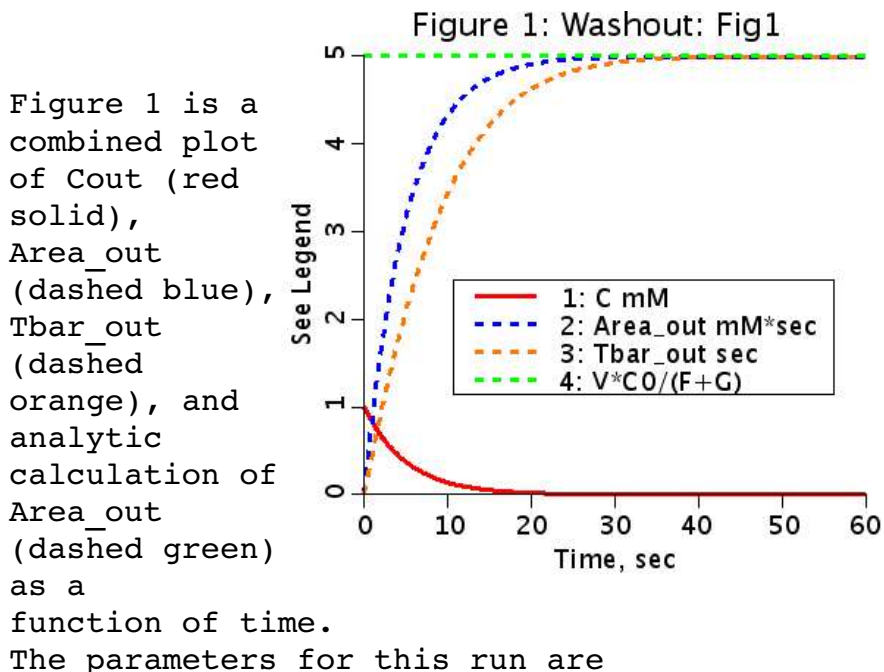
$$C(0) = C_0$$

For  $C_{in}$  constant, the analytic solution is

$$C_{analytic} = ((F \cdot C_{in}) - (F \cdot (C_{in} - C_0) - G \cdot C_0) \cdot \exp(-(F+G) \cdot t/V)) / (F+G)$$

## Figures and Explanations

Figure 1: Washout: Fig1 parameter set



$C_0 = 1$  mM (concentration in the compartment at time = 0),  
 $F = 0.01$  ml/sec (flow),  
 $V = 0.05$  ml (volume),  
 $G = 0$  ml/sec (clearance rate) and  
 $C_{in} = 0$  (the inflowing concentration).

The mean transit time through the chamber is  $V/F = 5$  seconds which equals the transit time of the outflow concentration since the transit time of the inflow concentration is zero. This means that the solution should fall to  $1/e$  of its original value in 5 seconds. Using the text button, we find that  $C(t=5 \text{ seconds}) = .36787944$  mM which is correct to all 8 digits.

The area of the outflow concentration approaches 5 mM\*sec (4.999693 mM\*sec) confirming that

$$Area_{out} = \int_0^{\infty} C(t) dt = \frac{V \cdot C_0}{F}, \text{ when } G=0.$$

Verify that when  $G > 0$ ,

$$Area_{out} = \int_0^{\infty} C(t) dt = \frac{V \cdot C_0}{F + G}$$

by setting  $C_0=2$  and  $G=0.03$ . The answer should be  $Area_{out} = 2.5$  mM\*sec.

Figure 2A: Concentrations: Fig2 parameter set  
 Figure 2B: Areas: Fig2 parameter set  
 Figure 2C: Area Ratio: Fig2 parameter set  
 Figure 2D: Tbar's: Fig2 parameter set

Figure 2A: Concentrations: Fig2

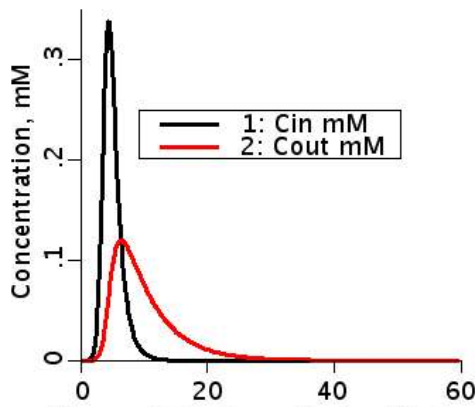


Figure 2B: Areas: Fig2

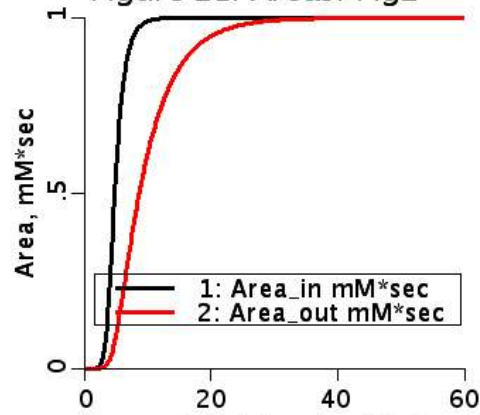


Figure 2C: Area Ratio: Fig2

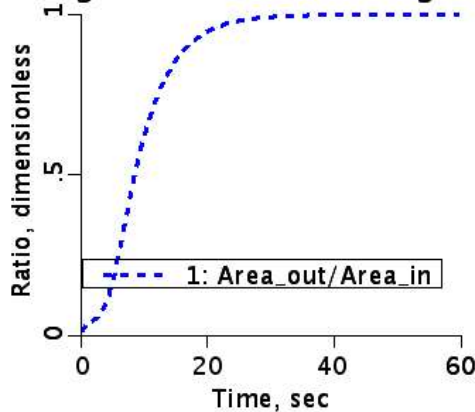
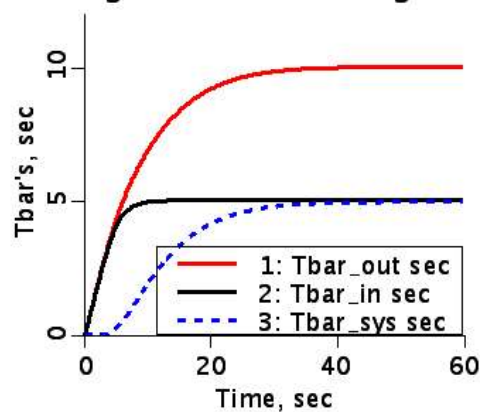


Figure 2D: Tbar's : Fig2



Load the Fig2 parameter set and run the model. The following parameters have been set:

$C_0$  = 0.0 mM (initial concentration at time = 0. )

$F$  = 0.01 ml/sec (flow),

$V$  = 0.05 ml (volume),

$G$  = 0 ml/sec (clearance rate) and

$C_{in}$  = The Lagged Normal input function use the function generator, `fgen_1`. (the inflowing concentration).

All plots are functions of time.

Figure 2A shows  $C_{in}$  and the more dispersed  $C_{out}$ .

Figure 2B shows the corresponding areas from integration. It is seen that both approach the same value.

Figure 2C shows the ratio of  $Area_{out}$  divided by  $Area_{in}$ . It shows that the ratio approaches 1 demonstrating that the integrated inflow concentration must equal the integrated outflow concentration in the absence of decay (clearance).

Figure 2D shows the transit times of  $C_{in}$ ,  $C_{out}$ , and the system. The transit time of the system approaches  $V/F = 5$  seconds.

Figure 3: Fraction Remaining and Consumed: Fig3 parameter set

The **black solid line** is the fraction consumed (or cleared). The **red solid line** is the fraction remaining.

Load the Fig3 parameter set and run the model. The following

parameters have been set:

$C_0 = 1.0$  mM (initial concentration at time = 0. )

$F = 0.00$  ml/sec (flow),

$V = 0.05$  ml (volume),

$G = 0.01$  ml/sec (clearance rate) and

$C_{in} = 0.0$  mM.

The time to reach  $C=C_0/e$  is 5 seconds as before, and the fraction of  $C$  consumed is  $C_0*(1-1/e)$ .

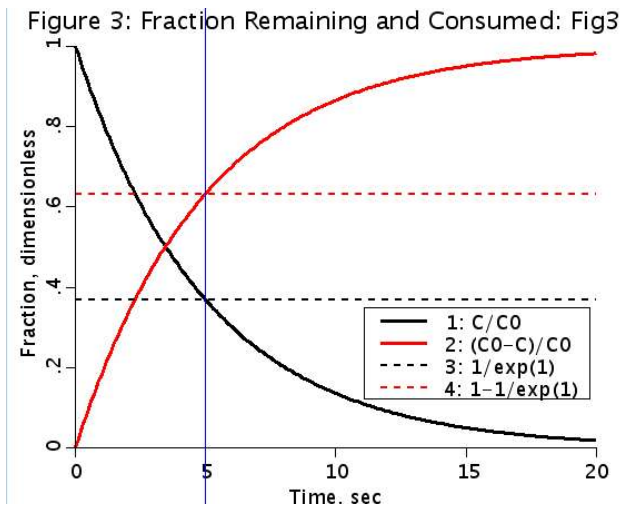


Figure 4: Steady State with  $C_{in}$  constant: Fig4 parameter set

$C_0=0.6$  mM,

$F = 0.01$

ml/sec,

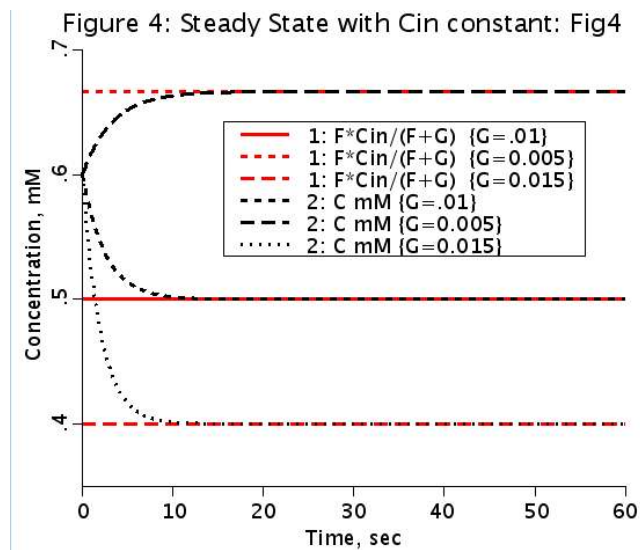
$G=0.01$

ml/sec,

$V=0.05$ ,

$C_{in} = 1.0$  mM

(constant).



The concentration of  $C(t)$  levels off at 0.5 mM. This is

because the clearance rate equals the flow so that half is consumed during the time that C is in the compartment.

Run loops with G=0.01, 0.005, and 0.015 ml/sec. The results are

G ml/sec	C(t=60) mM
0.005	0.666
0.010	0.500
0.015	0.400

The steady state solution for the equation

$$dC(t)/dt = (F/V)*(C_{in}-C(t))-(G/V)*C(t)$$

when  $C_{in}$  is constant is

$$C(t) = F*C_{in}/(F+G).$$

This value does not depend on  $C_0$ . Clicking on the show button for Curve 1, Figure 4, will reveal the expected value for this calculation.