Understanding the Comp2Exchange Model

This is a model of two compartments, one with volume V1 and initial concentration C10, and the other with volume V2 and concentration C20. The two compartments exchange material at a rate PS. PS stands for permeability surface area product. The exchange is passive and equal in both directions in this model. The governing ordinary differential equations are

$$dC_1/dt = (PS/V_1)*(C_2 - C_1) \text{ and}$$

$$dC_2/dt = (PS/V_2)*(C_1 - C_2) \ .$$

The initial conditions are given as

$$C_1(0) = C_{10} \text{ and } C_2(0) = C_{20} \ .$$

The analytic solutions are

$$C_{1\,analytic} = \frac{(C_{20}\cdot V_2 + C_{10}\cdot V_1 + V_2\cdot(C_{10} - C_{20})\cdot\exp(-PS\cdot(V_1+V_2)\cdot t/(V_1\cdot V_2)))}{(V_1+V_2)}$$

$$C_{2\,analytic} = \frac{(C_{20}\cdot V_2 + C_{10}\cdot V_1 - V_1\cdot(C_{10} - C_{20})\cdot\exp(-PS\cdot(V_1+V_2)\cdot t/(V_1\cdot V_2)))}{(V_1+V_2)} \ .$$

Questions:

(1) What are the steady state solutions?

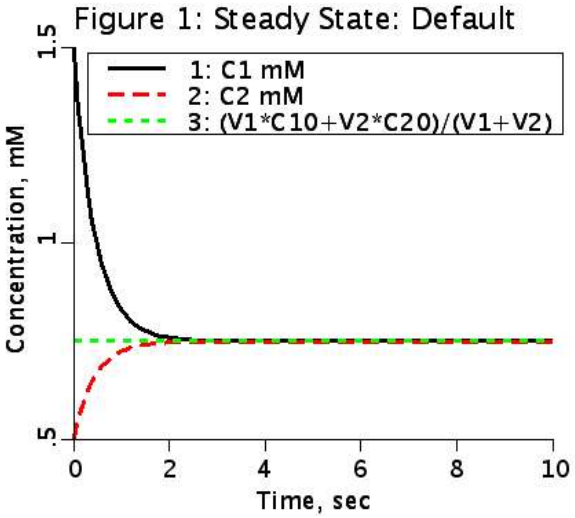
(2) If the volumes are decreased, does the time to reach equilibrium decrease, stay the same, or increase.

(3) If V1 is held constant and V2 becomes larger, what happens to C2 when (a) C10>C20? (b) C10=C20? (c) C10<C20?

(4) What is the flux of C from V1 to V2 when the steady state is reached? (Note: This is not a question about net flux.) What are the units for flux?

Figure and Explanation

Figure 1: Steady State: Default parameter set



The concentration C1, in compartment 1 with volume V1 is plotted (black solid line) as a function of time. The concentration C2, in compartment 2 with volume V2 is also plotted (red dashed line). The steady state concentrations are calculated for all time (green dashed line).

The steady-state concentrations are calculated using one of the ordinary differential equations and the mass conservation statement. Note how the volumes of the two compartments are included in a mass conservation statement.

dC1/dt = 0 = (PS/V1)*(C2-C1) => C1=C2.

and solving the mass balance equation,

V1*C1+ V2*C2 = V1*C10+V2*C20 becomes

$$C1 = C2 = \frac{(V1*C10+V2*C20)}{(V1+V2)} .$$

Use LOOPS to answer Questions 2 and 3, and explain your results.