Understanding the Comp2FlowExchangeReaction Model

This is a two compartment model (plasma and isf) exchange model with flow in the plasma compartment. Both spaces are instantaneously well mixed. A and B reversibly convert to each other in the isf space. The isf space can also be used as a cell space. Flow, Fp, and exchange rates, PSa and PSb, have the same units, ml/(g*min) (milliliters per minute per gram of tissue). These units are used in the physiological terminology to relate them to fluxes per gram of tissue.

The conversion rates have the same units, ml/(g*min). Ga2b is the conversion rate of A going to B, and Gb2a is the conversion rate of B becoming A.

The steady state solutions for constant inflow of A are solved implicitly, using the final value of the input concentration (assumed to have been constant).

The governing ordinary differential equations are

$$dA_p/dt = (F_p/V_p)\cdot(A_{\mathrm{in}} - A_p) + (PS_a/V_p)\cdot(A_{isf} - A_p),$$

$$dB_p/dt = (F_p/V_p)\cdot(\quad - B_p) + (PS_b/V_p)\cdot(B_{isf} - B_p),$$

$$dA_{isf}/dt = (PS_a/V_{isf})\cdot(A_p - A_{isf}) - (G_{a2b}/V_{isf})\cdot A_{isf} + (G_{b2a}/V_{isf})\cdot B_{isf},$$

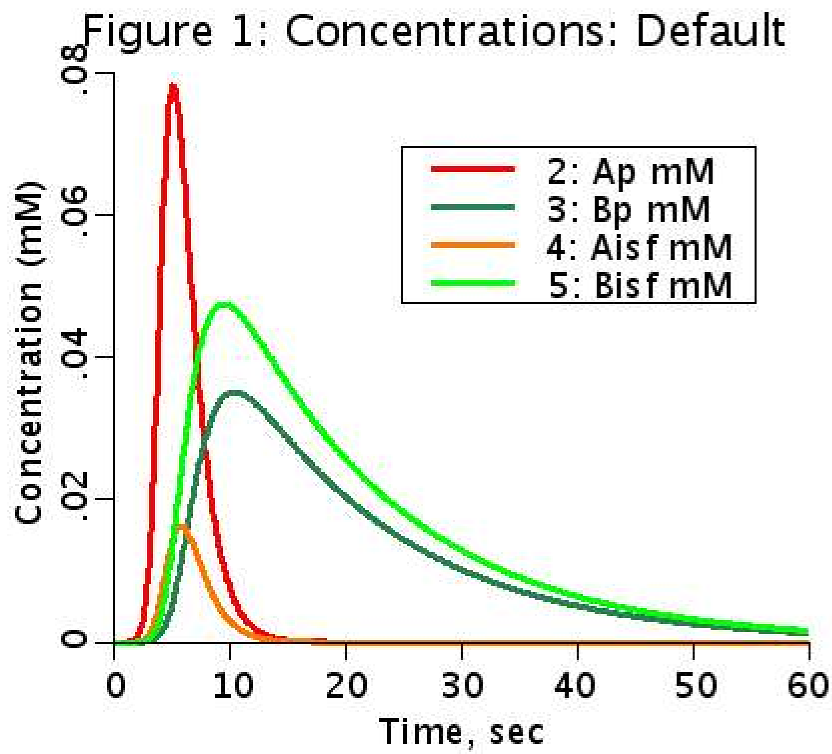
$$dB_{isf}/dt = (PS_b/V_{isf})\cdot(B_p - B_{isf}) + (G_{a2b}/V_{isf})\cdot A_{isf} - (G_{b2a}/V_{isf})\cdot B_{isf}.$$

The initial conditions are given as

$$A_p(0) = A_p 0,\ A_{isf}(0) = A_{isf} 0,\ B_p(0) = B_p 0,\ \text{and}\ B_{isf}(0) = B_{isf} 0.$$

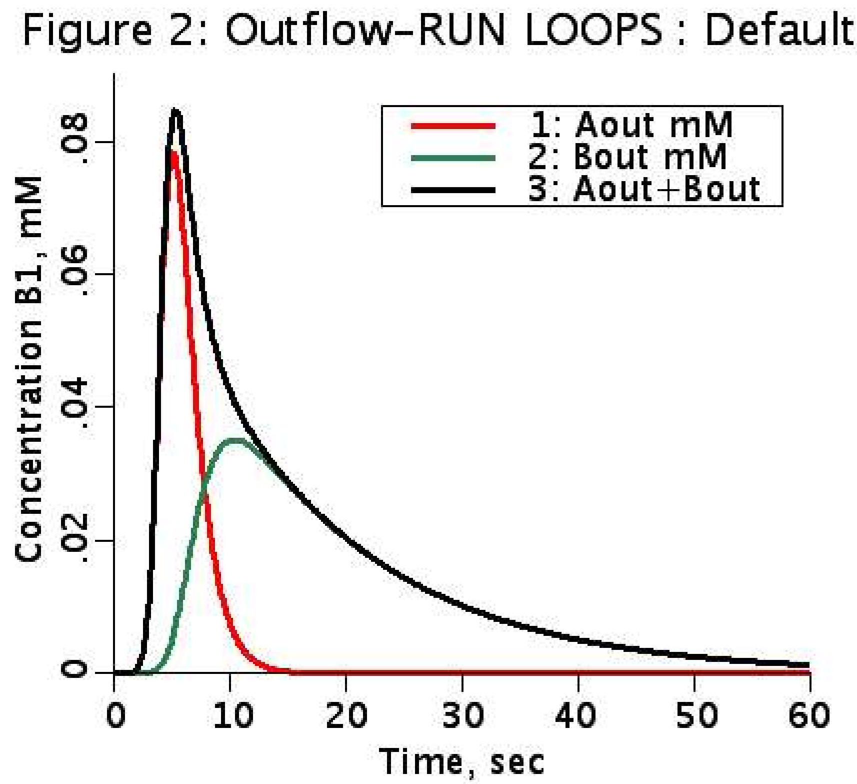
Figures and Explanations

Figure 1: Concentrations: Default parameter set



A lagged normal input curve is used as the inflow concentration, Ain. The concentration of Ap (A in the plasma, red), Bp (B in the plasma, dark green), Aisf (A in the isf, orange), and Bisf (B in the isf, light green) are plotted as functions of time.

Figure 2: Outflow-RUN LOOPS : Default parameter set



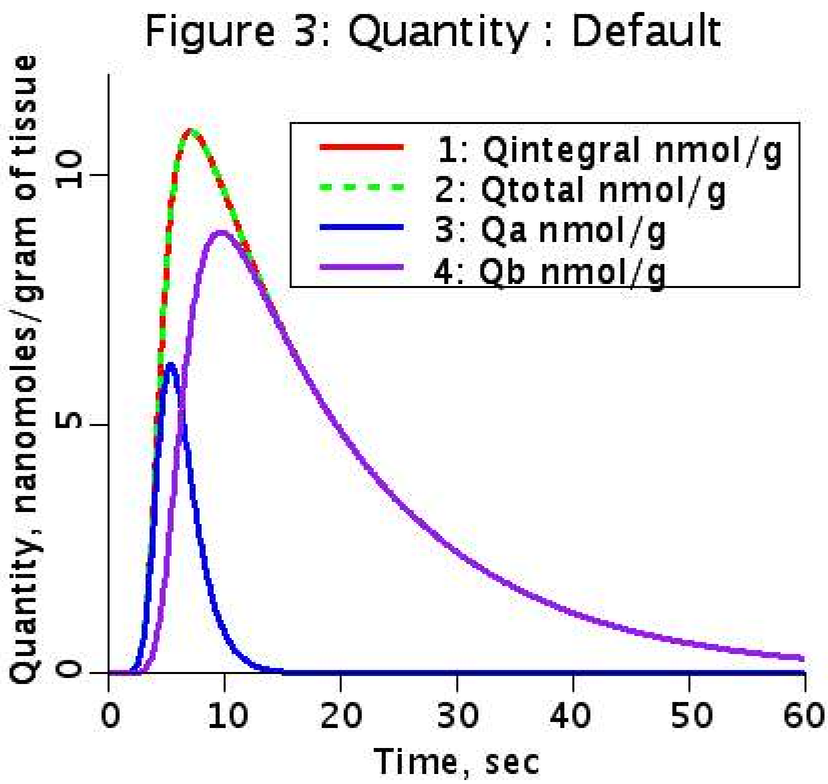
The plasma compartment concentration outflow of A (red) and B (dark green) and their sum (Aout+Bout) are plotted as a function of time.

What happens to the sum of the outflows if the conversion rate of A to b, Ga2b, is increased by a factor of 100. Go to the loops page and run to find out. What happens if the reverse reaction is turned on?

Even though the conversion of A to B has been increased from 10 ml/(g*min) t0 1000 ml/(g*min), the sum of the outflow concentrations did not change. This is because PSa=PSb. You can demonstrate this by setting PSb to 0.1 and running loops again.

What happens if the reverse reaction is turned on? Turn the outer Loop Configurator to auto and run again. If PSa equals PSb, you should see no change in the sum of Aout+Bout.

Figure 3: Quantity: Default parameter set



The total amount of material in the system is calculated in two different ways:

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   Qtotal(t) = Vp*(Ap(t)+Bp(t)) + Visf*(Aisf(t)+Bisf(t))
 (red line)

   and
                  /t
   Qintegral(t)= | Fp*(Ain(t')-Aout(t')-Bout(t') dt'
                /0
 (dashed green line).
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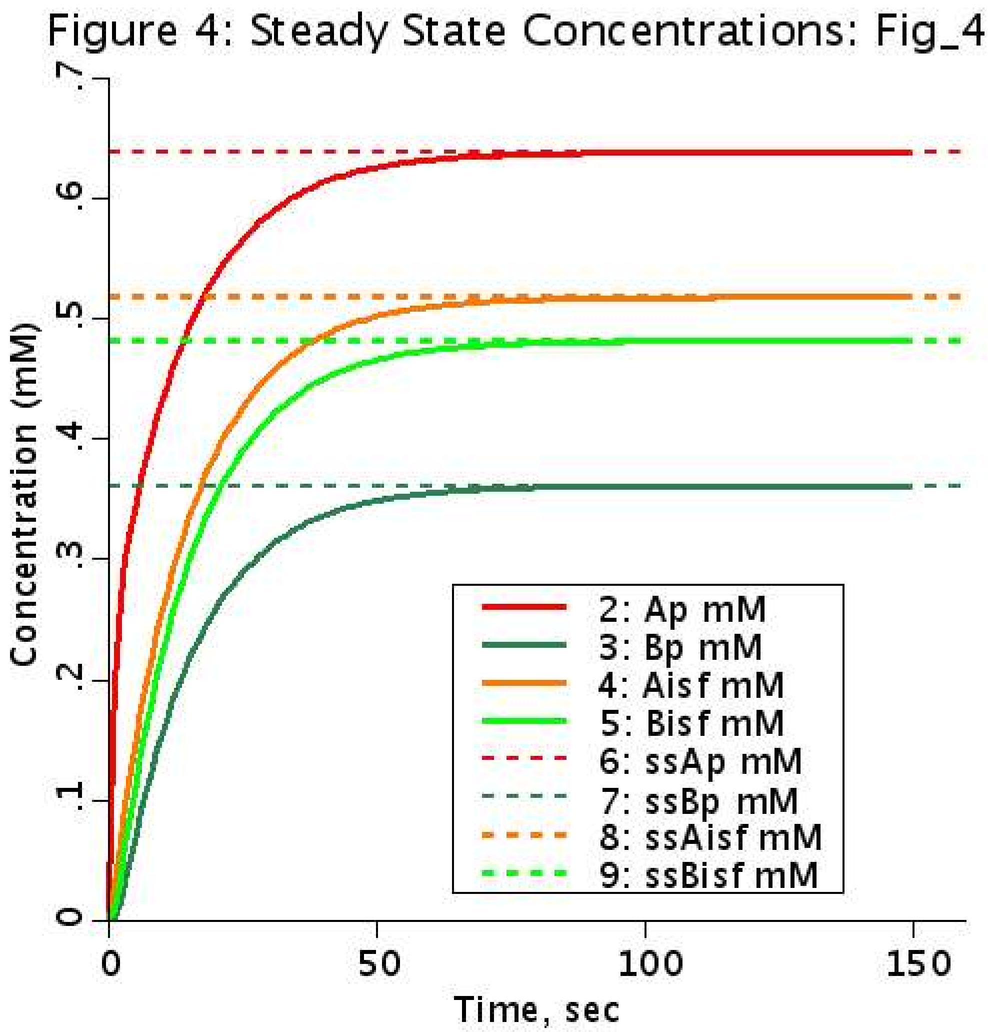
Because the answers are the same, in the plot they appear as a single dashed red and green line.

Run loops to see that the rate constants for A becoming B and B becoming A have no effect on the total amount of the combined material in the system.

Figure 4: Steady State Concentrations: Fig_4 parameter set



The model is given a constant input, Ain = 1, and run to a steady state. The steady state concentrations are also calculated from a set of implicit equations which come from the ordinary differential equations.
If the set of ordinary equations is given by

$$d(\overline{X}).dt = F(\overline{X}), \text{ where } \overline{X} \text{ is a vector, then}$$

$\overline{0}=F(\overline{X})$ is a system of implicit equations which can be solved for $\overline{X}$.

The implicit variables for Ap, Bp, Aisf, and Bisf have been named ssAp, ssBp, ssAisf, and ssBisf.