

Sensitivity Functions

Consider the single compartment with flow model. The governing equation is

$$\frac{d}{dt} C(t) = \frac{F (C_{in}(t) - C(t))}{V}. \text{ Let the initial condition, } C(0)=I \text{ and } C_{in}(t)=0.$$

The solution is

$$C(t) = \exp\left(-\frac{F \cdot t}{V}\right).$$

The sensitivity functions are the derivatives of $C(t)$ with respect to the parameters F and V .

$$\frac{d}{dF} C(t) = C(t) \cdot \left(-\frac{t}{V}\right).$$

$$\frac{d}{dV} C(t) = C(t) \cdot \left(-\frac{F \cdot t}{V^2}\right).$$

The ratio of the two sensitivity functions is

$$\frac{\frac{d}{dV} C(t)}{\frac{d}{dF} C(t)} = -\frac{F}{V}, \text{ which shows that they only differ by a multiplying constant.}$$

If $C_{in}(t)$ is some complicated function, the ratio of the sensitivity functions is still the same ratio.

What does the sensitivity function mean? Where the sensitivity function is negative, it means that increasing the value of a parameter will decrease the value of the variable at that point. Where the sensitivity function is positive, it means that increasing the value of the parameter will increase the value of the variable at that point. Where the absolute value of the sensitivity function is maximum, the value of the variable will change the most. Hence sensitivity functions tell us which parameters affect the peak and tail of a measurement curve and whether we should increase parameters or decrease them to fit the data.

Question: Is it useful to optimize with parameters with similar sensitivity functions?