

Crone-Renkin Expression

The Crone-Renkin expression relates PS (permeability multiplied by surface area) to F (flow) and extraction.

Consider the equations for a two region model with two tracers (plasma and permeant) and no diffusion. Assume there is no back flux from the second region for the permeant tracer. Then the partial differential equations for the two tracers are given by

$$\partial/\partial t C_{ref} = -(F * L/V) \partial/\partial x C_{ref}$$

$$\partial/\partial t C_{permeant} = -(F * L/V) \partial/\partial x C_{permeant} - (PS/V) C_{permeant}.$$

Assume a constant inflow, C_0 . The steady state equations become

$$\partial/\partial x C_{ref} = 0$$

$$(\partial/\partial x C_{permeant})/C_{permeant} = -PS/(F * L).$$

The solutions are

$$C_{ref} = C_0$$

$$C_{permeant} = C_0 \exp^{(-PSx/(FL))}$$

The extraction, E , is given by

$$E = (C_{ref} - C_{permeant})/C_{ref} \text{ at } x = L.$$

Therefore,

$$E = 1 - \exp^{(-PSL/(FL))}.$$

This can be rewritten as

$$\exp^{(-PS/F)} = 1 - E.$$

Taking the natural logarithm of both sides

$$-PS/F = \log_e (1 - E)$$

which is usually written as

$$PS = -F \log_e (1 - E).$$

In indicator dilution experiments with multiple tracers F is measured. We don't normally calculate the PS from the extraction curves by using the maximum extraction because flow is heterogeneous and there is back flux from the interstitial fluid region to the capillary.

What happens if the permeant tracer in the plasma can exchange with both the interstitial fluid region and the endothelial cells. Work out the solution when the governing equation for the steady state equation for the permeant tracer is given by

$$\partial/\partial t C_{permeant} = -(F * L/V) \partial/\partial x C_{permeant} - (PSg/V) C_{permeant} - (PSecl/V) C_{permeant}.$$

Answer on backside.

$$PSg + PSecl = -F \log_e (1 - E).$$