Crone-Renkin Expression

The Crone-Renkin expression relates PS (permeability multiplied by surface area) to F (flow) and extraction.

Consider the equations for a two region model with two tracers (plasma and permeant) and no diffusion. Assume that there is no back flux from the second compartment for the permeant tracer. Then the steady state equations are given by

$$0 = -\frac{F}{V}\cdot\left(\frac{d}{dx}C_{ref}\right), \text{ and}$$
$$0 = -\frac{F\cdot L}{V}\cdot\left(\frac{\partial}{\partial x}C_{permeant}\right) - \frac{PS}{V}\cdot C_{permeant}.$$

Let the steady-state inflow be C_0.
The the solutions are

$$C_{ref} = \mathrm{C}_0, \text{ and } C_{permeant} = C_0 e^{-\frac{PSx}{F\cdot L}}.$$

The extraction, E, is given by
$$E = \frac{C_{ref} - C_{permeant}}{C_{ref}} \text{ at x=L.}$$
Therefore
$$E = 1 - e^{-\frac{PSL}{F\cdot L}}.$$

This equation can be rewritten as

$e^{-\frac{PS}{F}} = 1 - E$. Taking the natural logarithm of both sides
$-\frac{PS}{F} = \log_e(1 - E)$ which is usually written as

$$PS = -F\log_e(1 - E)$$

In indicator dilution experiments with multiple tracers, we calculate the PS from the extraction curves by using the maximum extraction. Since F is usually measured, we can calculate PS.

What happens if the tracer can exchange with both the interstitial fluid region isf) and the endothelial cells from the plasma space? Work out the solution when the governing steady state equation for the permeant tracer is given by
$$0 = -\frac{F\,L}{V}\left(\frac{\partial}{\partial x}C_{permeant}\right) - \frac{PS_g}{V}\cdot C_{permeant} - \frac{PS_{ecl}}{V}\cdot C_{permeant} \text{ for } PS_{ecl}.$$
when there is a plasma tracer, an extracellular (enters the plasma and isf) tracer, and a permeant tracer.